CHAPTER 7
LINEAR MOMENUTUM

The Law of Conservation of Energy is one of four conservation laws. The other three include: The Law of Conservation of linear momentum, angular momentum and electric charge.

7.1 MOMENTUM AND ITS RELATIONSHIP TO FORCE

- **linear momentum**—defined as the product of mass x velocity. Plural is momenta and short is  symbolized by "p"; I have no idea why!

- not to be confused with either  or 

$$\mathbf{p = mv} \quad \text{and its units are kg} \cdot \text{m/s}$$

- Momentum is a vector since v is a vector and it is multiplied by a scalar. That means that momentum has a direction that is in the direction of the velocity.
- Consider the p of an 18-wheeler versus an electron. Lots of mass with slow speed compared to very little mass but lots of speed! Both can fairly easily penetrate a few inches of lead!
- The more p, the harder it is to stop an object & the greater the effect of stopping the object!
- F IS required to Δp in magnitude OR direction
- Restate Newton's 2nd Law:
  The rate of change of momentum is equal to the net force applied to it so….

$$\sum \mathbf{F = ma} = \frac{\Delta \mathbf{p}}{\mathbf{t}}$$

Humor me,  and watch this…...

$$\Sigma F = \frac{\Delta p}{\Delta t} = \frac{mv - mv_o}{\Delta t} = \frac{m(v - v_o)}{\Delta t}$$

$$= m\frac{\Delta v}{\Delta t}$$

$$= ma$$

Look familiar??? We'll keep using this idea, so come to appreciate it!

Example 7.1

Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it. (That is we ignore any splashing back.) What is the force exerted by the car?

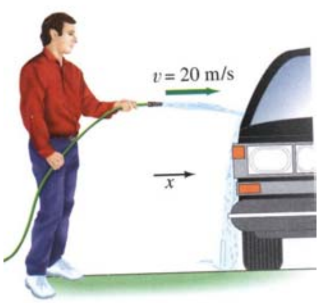


FIGURE 7–1 Example 7–1.

Example 7.2
What if the water splashed back from the car? Would the force on the car be greater or less?

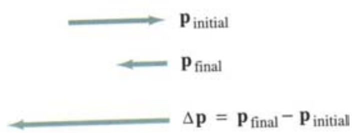
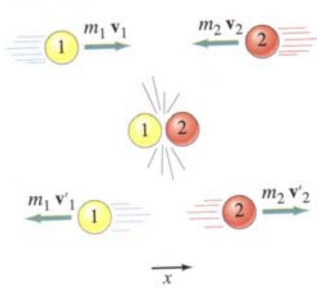


FIGURE 7–2 Conceptual Example 7–2. Momentum of water before and after splashing back, and $\Delta\mathbf{p}$.

7.2 CONSERVATION OF LINEAR

FIGURE 7–3 Momentum is conserved in a collision of two balls.



The vector sum of two colliding objects remains constant. Consider the head-on collision of two billiard balls:

- Assume the net *external* force equals zero on the system: $F_{net} = 0$.
- The only significant forces are those that each ball exerts on the other during the collision.
- The ***sum*** of their momenta is found to be the same BEFORE as AFTER the collision.

BEFORE =

AFTER

$$\textbf{p BEFORE} = \textbf{p AFTER}$$
$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

The v' variable indicates that the velocity changed as a result of the collision.

- THE TOTAL VECTOR SUM OF THE SYSTEM OF 2 BALLS IS CONSERVED: it stays constant!
- How is this related to Newton's Laws of Motion?
    - Assume that the F one ball exerts on the other is constant during the collision which occurs over a short time interval,)t

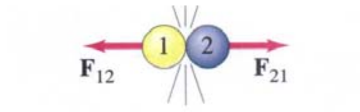


FIGURE 7–4 Forces on the balls during the collision of Fig. 7–3.

$F = \frac{)p}{)t}$ so . . . . .

$)p = F)t$

apply this to ball 2 using + x-direction to be to the right:

$)p = m_2v'_2 - m_2v_2 = F_{2\ on\ 1})t$ AND Newton's 3[rd] Law states $F_{1\ on2} = -F_{2\ on\ 1}$

apply this to ball 1

$)p = m_1v'_1 - m_1v_1 = F_{1\ on\ 2})t = -F_{2\ on\ 1})t$

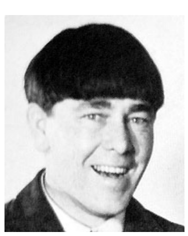
we can combine these 2 equations together (their right sides differ only by a minus sign)

$m_1v'_1 - m_1v_1 = -(m_2v'_2 - m_2v_2)$

$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$

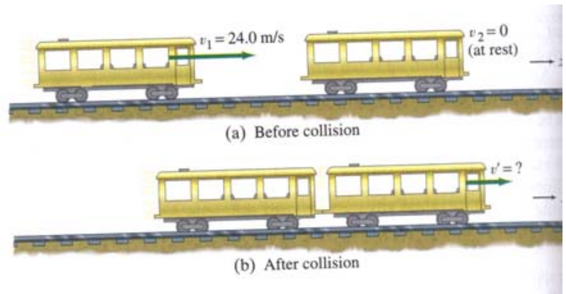
This also works on more than just 2 bodies colliding!

THE TOTAL MOMENTUM OF AN ISOLATED SYSTEM OF BODIES REMAINS CONSTANT!!!

- **<u>Isolated</u>**-- means the only forces present are those between the objects of the system
- $E\ F_{external} = 0$ due to Newton's 3[rd] Law
-  WON"T be conserved IF external forces are present whose vectorial sum is NOT equal to zero!
    - a falling rock does NOT conserve p since F_g (an excellent example of an external force) is acting on it and changing its p.
- However, if we include the Earth in the system, p_{total} of the rock and the Earth is conserved

- Of course, this means the Earth comes up to meet the ball. Try not to panic—the Earth's mass is SO huge, its upward velocity is very tiny!

Example 7.3
A 10,000 kg railroad car traveling at a speed of 24.0 m/s strikes an identical car at rest. If the cars lock together as a result of the collision, what is their common speed afterward?



The Law of Conservation of Momentum is particularly useful when dealing with collisions (ballistics) & explosions or rocket propulsions—a form of a controlled explosion!

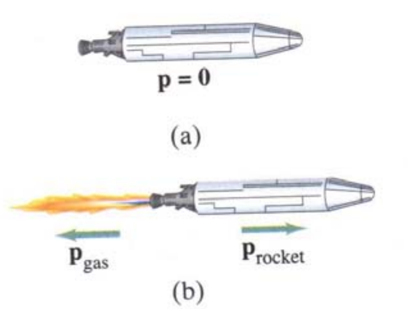


FIGURE 7–6 (a) A rocket, containing fuel, at rest in some reference frame. (b) In the same reference frame, the rocket fires and gases are expelled at high speed out the rear. The total vector momentum, $\mathbf{p}_{gas} + \mathbf{p}_{rocket}$, remains zero.

- Before a rocket is fired, p_T [the fuel + rocket's momenta] = 0.
  - As fuel burns, p_T remains unchanged
  - The backward p of the expelled gas is just balanced by the forward p gained by the rocket!
  - Thus, rockets can accelerate in empty space—no need for the expelled gasses to push against Earth or air molecules like they must @ launch.
  - The recoil of a gun being fired or throwing a package from a boat are similar examples.

Example 7.4
Calculate the recoil velocity of a 5.0 kg rifle that shoots a 0.050 kg bullet at a speed of 120 m/s.

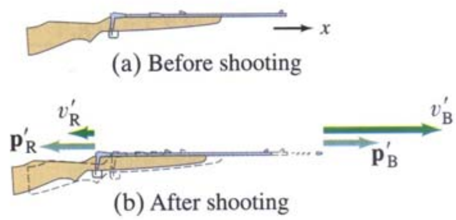


FIGURE 7–7 Example 7–4.

7.3 COLLISIONS AND IMPULSE

I can't resist this moment to tell you that at the subatomic level, scientists learn about structure of nuclei and atomic constituents and about the nature of forces involved by careful study of collisions between nuclei and or elementary particles. I'm still bitter about the dismantling of the super conducting super collider ! I was looking forward to years of field trips!

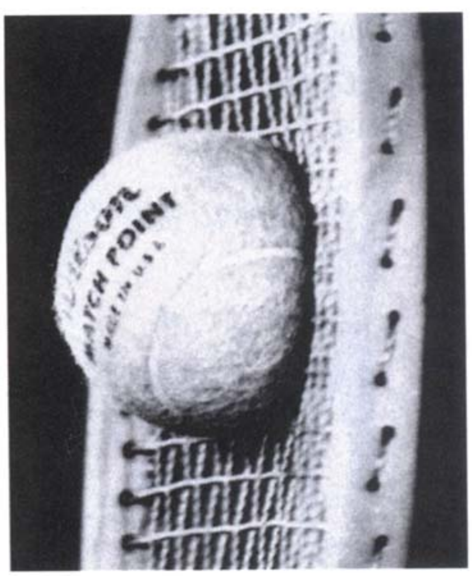
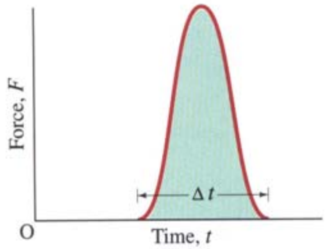


FIGURE 7–8 Tennis racket striking a ball. Note the deformation of both ball and racket due to the large force each exerts on the other.

When a collision occurs between 2 objects that are not overly rigid, both objects are deformed—all objects are deformed at the atomic level since an atom is mostly empty space!

FIGURE 7–9 Force as a function of time during a typical collision.



Both particles are deformed in a collision—at the moment of impact, F jumps from ZERO to a very large value in a very short time and abruptly returns to ZERO again.

- A graph of the F an object exerts on another object during a collision as a function of time, is something like that shown by the red curve in fig. 7-9.
-)t is very small!

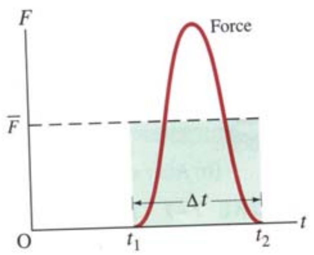


FIGURE 7–10 The average force $\overline{F}$ acting over an interval of time Δt gives the same impulse ($\overline{F}\,\Delta t$) as the actual force.

F =)p /)t   applies to EACH of the objects in a collision

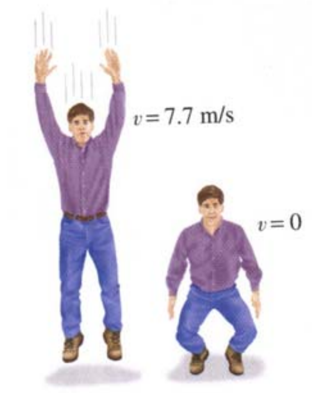
F)t = Impulse =)p

- The TOTAL)p is the impulse in a collision
- Impulse is of help mainly when dealing with forces that act over a short time, hitting a baseball with a bat…stuff like that.
- The F is often NOT constant but varies like the graph in figure 7-10 above.
- It is sufficient to approximate such a varying F by using F of a time)t, like the dashed line in the graph.
- F is chosen so that the shaded area (F x)t) = area under the *actual curve* F vs. t which represents the impulse.
- NOTE: the same impulse and same change in momentum ()p), can be given to an object by a smaller force, F, if applied over a greater time, as long as F)t remains the same.

Exercise 7.5

a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m.

Then estimate the average force exerted on the person's feet by the ground, if the landing is



Free-body diagrams are always useful!

b) stiff-legged

FIGURE 7–12 When the person lands on the ground, the average net force during impact is $\bar{F} = F_{grd} - mg$, where F_{grd} is the force the ground exerts upward on the person.

c) with bent legs. In the former case, assume the body moves 1.0 cm during impact, and in the second case, when the legs are bent, around 50 cm.

7.4 CONSERVATION OF ENERGY AND MOMENTUM IN COLLISIONS

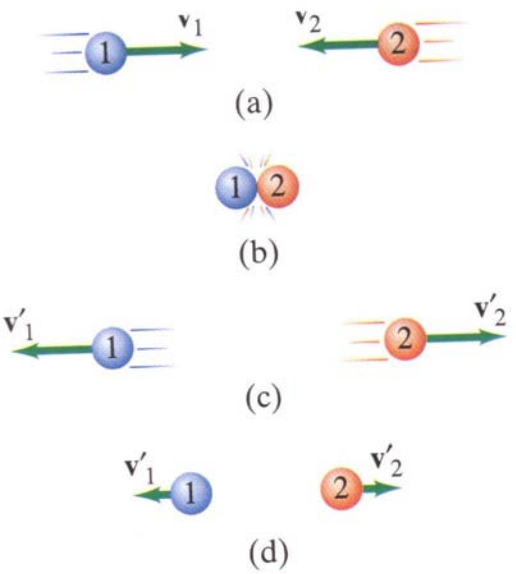


FIGURE 7–13 Two equal mass objects (a) approach each other with equal speeds, (b) collide, and then (c) bounce off with equal speeds in the opposite directions if the collision is elastic, or (d) bounce back much less or not at all, if the collision is inelastic.

If a collision occurs between two hard objects with no heat produced in the collision, then KE is also conserved. Of course, for the brief moment during which the 2 objects are in contact, some [or all] of the E is stored in the form of elastic PE.

- Such a collision, with conservation of KE, is defined as **<u>an elastic collision</u>** (recall the kinetic theory of matter and its postulates?)
- $\frac{1}{2}\ m_1v_1^2 + \frac{1}{2}\ m_2v_2^2 = \frac{1}{2}\ m_1v'^2_1 + \frac{1}{2}\ m_2v'^2_2$

- I can't resist this chemistry/atomic moment
  - Although you may not see damage in an elastic collision macroscopically, elastic collisions at the atomic level are an ideal that is never reached—such as in an ideal gas or the kinetic theory of matter.
  - Thermal, sound, etc. energy is always produced
  - Even with these "neglects", the Total Energy is conserved once they are considered.

7.5 ELASTIC COLLISIONS IN ONE DIMENSION--SOLVING PROBLEMS USING ENERGY AND MOMENTUM CONSERVATON

Let's start with an elastic collision between two small particles head-on, so all motion is along a straight line:

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

AND … $$\tfrac{1}{2} m_1v_1^2 + \tfrac{1}{2} m_2v_2^2 = \tfrac{1}{2} m_1v'^2_1 + \tfrac{1}{2} m_2v'^2_2$$

My favorite! Two equations so we can solve for 2 unknowns!
A useful result:

$$m_1 (v_1 - v'_1) = m_2 (v'_2 - v_2) \qquad \text{(i)}$$

and re-write the KE equation as (canceling out ½'s and rearranging)

$$m_1 (v_1^2 - v'^2_1) = m_2 (v'^2_2 - v_2^2)$$

or [noting that $(a-b)(a+b) = a^2 - b^2$] we write this as

$$m_1 (v_1 - v'_1)(v_1 + v'_1) = m_2 (v'_2 - v_2)(v'_2 + v_2) \qquad \text{(ii)}$$

Divide equation (ii) by equation (i) and assuming there was a change in velocity we obtain

$$(v_1 + v'_1) = (v'_2 + v_2)$$

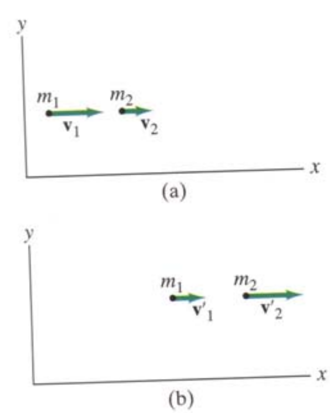
We can rewrite this equation as:

$$v_1 - v_2 = v'_2 - v'_1$$

$$= -(v'_1 - v'_2)$$

Translation? For any elastic AND head-on collision, the relative speed of 2 particles after collision has the same magnitude as before (but opposite direction), <u>regardless of mass!!</u>

FIGURE 7–14 Two particles, of masses m_1 and m_2, (a) before the collision, and (b) after the collision.



Example 7.6
A billiard ball of mass m moving with speed v collides head-on with a second ball of equal mass at rest. What are the speeds of the two balls after the collision, assuming it is elastic?

Example 7.7
A proton traveling with mass of 1.01 u (unified atomic mass units) traveling with a speed of 3.60×10^4 m/s has an elastic head-on collision with a He nucleus. (mass of He = 4.00u) initiall at rest [tough for a gas molecule!]. What are the velocities of the proton and helium nucleus after the collision?

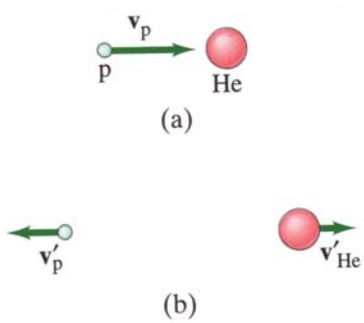


FIGURE 7–16 Example 7–7: (a) before collision, (b) after collision.

7.6 INELASTIC COLLISIONS

- **<u>Inelastic collisions—</u>**KE is NOT conserved

  $KE_1 + KE_2 = KE'_1 + KE'_2 +$ thermal E & other forms of E
- **<u>Completely inelastic collisions</u>**—ones where the 2 objects stick together and move as one new mass! The total E is always conserved; the maximum amount of KE is converted into another form of energy in completely inelastic collisions.

Example 7.8
For the completely inelastic collision of two rail road cars that we considered in Example 7.3, calculate how much of the initial DE is transformed to thermal or other forms of energy.

Example 7.9
The *ballistic pendulum* is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass m, is fired into a large block (of wood or other material) of mass M, which is suspended like a pendulum. (Usually, M is somewhat greater than m.) As a result of the collision, the pendulum-projectile system swings up to a maximum height, h. Determine the relationship between the initial speed of the projectile, v and the height h.

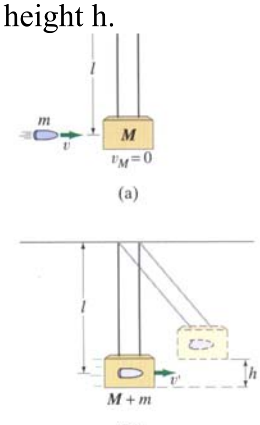


FIGURE 7–17 Ballistic pendulum (Example 7–9).

7.7 COLLISIONS IN TWO OR THREE DIMENSIONS

Conservation of E and momentum can also be applied to collisions in 2 or 3-D. A favorite example is when on particle, the projectile, strikes a target particle at rest.

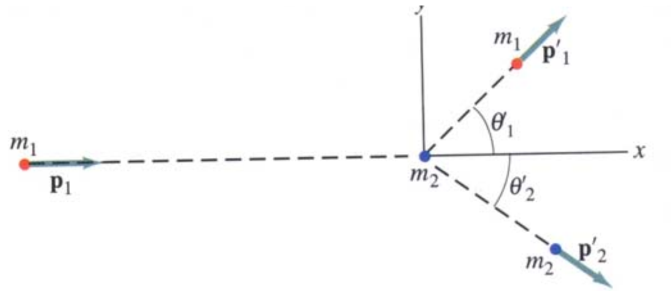


FIGURE 7–18 Particle 1, the projectile, collides with particle 2, the target. They move off, after the collision, with momenta $\mathbf{p}'_1$ and $\mathbf{p}'_2$ at angles θ'_1 and θ'_2.

This figure shows particle 1 heading along the x axis toward particle 2 which is initially at rest. When 1 hits its target, both go off at respective angles. This is complicated by the presence of electric or magnetic fields

We choose the xy plane to be the plane in which the initial and final momenta lie.

Because momentum is a vector and is conserved, its components in the x and y directions remain constant.

In the x…

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}$$

or

$$m_1 v_1 = m_1 v'_1 \cos 2'_1 + m_2 v'_2 \cos 2'_2$$

Because there is no initial motion in the y direction, the y component of the total momentum is ZERO:

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$$

or

$$0 = m_1 v'_1 \sin 2'_1 + m_2 v'_2 \sin 2'_2$$

Example 7.10
A billiard ball moving with speed $v_1 = 3.0$ m/s in the +x direction strikes an equal-mass ball initially at rest. The two balls are observed to move off at 45° , ball 1 above the x axis and ball 2 below. This $2'_1 = 45°$ and $2'_2 = -45°$. What are the speeds of the two balls?

Here we go again! 2 equations so I can solve for 2 unknowns!

If we know a third equation……we can solve for a third unknown!

$$KE_1 + KE_2 = KE'_1 + KE'_2$$

OR for the collision shown in figure 7.18

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v'^2_1 + \tfrac{1}{2} m_2 v'^2_2$$

➥ PROBLEM SOLVING Momentum Conservation and Collisions

1. Be sure no significant external force acts on your chosen system. That is, the forces that act between the interacting bodies must be the only significant ones if momentum conservation is to be used. [Note: If this is valid for a portion of the problem, you can use momentum conservation for that portion only.]
2. Draw a diagram of the initial situation, just before the interaction (collision, explosion) takes place, and represent the momentum of each object with an arrow and label. Do the same for the final situation, just after the interaction.
3. Choose a coordinate system and "+" and "−" directions. (For a head-on collision, you will need only an x axis.) It is often convenient to choose the $+x$ axis in the direction of one object's initial velocity.
4. Write momentum conservation equation(s):

   total initial momentum = total final momentum.

   You have one equation for each component (x, y, z); only one equation for a head-on collision. [Don't forget that it is the *total* momentum, not the individual momenta, that is conserved.]
5. If the collision is elastic, you can also write down a conservation of kinetic energy equation:

   total initial KE = total final KE.

   [Alternately, you could use Eq. 7–7: $v_1 - v_2 = v'_2 - v'_1$, if the collision is one dimensional (head-on).]
6. Solve algebraically for the unknown(s).

7.8 CENTER OF MASS (CM)

The diver in a under goes only translational motion while the diver in b undergoes BOTH translational motion and rotational motion.

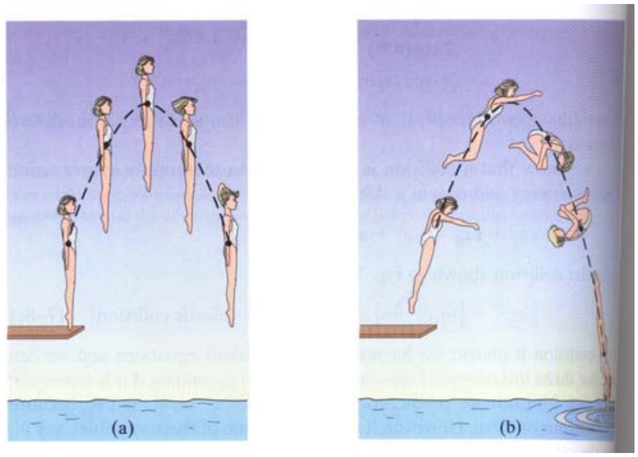


FIGURE 7–19 The motion of the diver is pure translation in (a), but is translation plus rotation in (b).

- **<u>center of mass</u>**—the one point that moves in the same path that a particle would if subjected to the same net force.

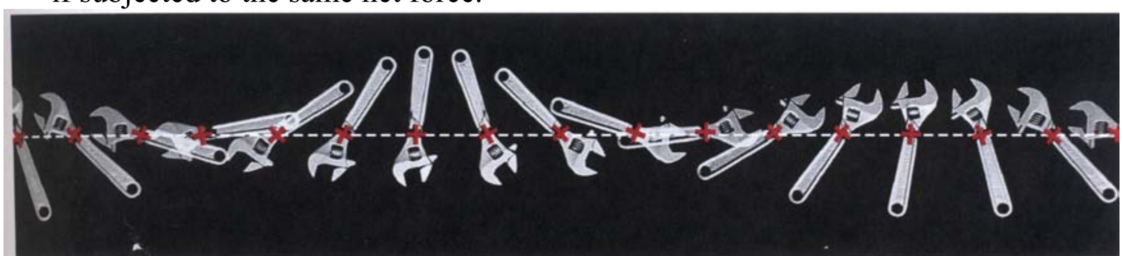


FIGURE 7–20 Translation plus rotation: a wrench moving over a horizontal surface. The CM, marked with a +, moves in a straight line.

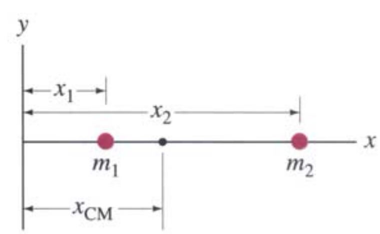


FIGURE 7–21 The center of mass of a two-particle system lies on the line joining the two masses.

Center of mass (x coordinate)

- The general motion of an extended body [or system of bodies] can be considered as the sum of the translational motion of the CM, plus rotational, vibrational, or other types of motion about the CM.
- Look again at the diver. She follows a parabolic path whether she rotates or not.

- We consider any extended body as being made up of many tiny particles BUT we are going to cheat and focus on only 2 selected particles first.
- Next, we select an x coordinate system to make our lives easy!

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1x_1 + m_2x_2}{M}$$

It's even easier if the masses are equal

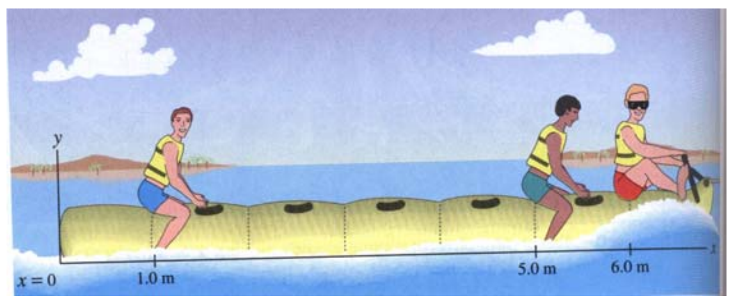
$$x_{CM} = \frac{m(x_1 + x_2)}{2m} = \frac{(x_1 + x_2)}{2}$$

But common sense probably told you that!

If one mass is greater than the other, the CM is closer to the larger mass. Can have more than 2 terms!

Example 7.11

Three people of roughly equivalent mass, m, on a lightweight banana boat sit along the x axis at positions $x_1 = 1.0$ m $x_2 = 5.0$ m and $x_3 = 6.0$ m. Find the position of the CM.



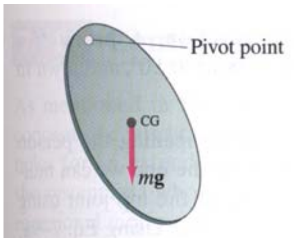


FIGURE 7–23 The force of gravity, considered to act at the CG, causes the body to rotate about the pivot point unless the CG is on a vertical line directly below the pivot, in which case the body remains at rest.

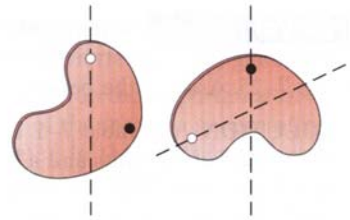
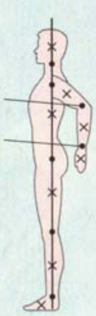


FIGURE 7–24 Finding the CG.

- If the particles are spread out in two or three dimensions, then we need to specify not only the x coordinates but also the y and z coordinates.
- Just sub y's in for the x's
- **center of gravity**—the point on a body at which the force of gravity can be considered to act—there is a subtle difference between CG and CM, but they are generally the same unless you're an elephant or other extremely large irregularly shaped body!
- Easier to determine experimentally rather than analytically.
  - Suspend a body and it will swing UNLESS its CG lies on the vertical directly below the point from which it is suspended.
  - Hang the object if 2-D [flat; one plane] from 2 different pivot points and draw the 2 plumb lines, their intersection is the CG.
  - 3-D objects require 3 suspension points so that their plumb lines DO NOT lie in the same plane.
  - Uniform cylinders (wheels), spheres, and rectangular solids have a CG at their geometric center.

TABLE 7–1
Center of Mass of Parts of Typical Human Body (full height and mass = 100 units)

Distance of Hinge Points (%)	Hinge Points (•) (Joints)	Center of Mass (×) (% Height Above Floor)		Percent Mass
91.2	Base of skull on spine	Head	93.5	6.9
81.2	Shoulder joint	Trunk and neck	71.1	46.1
		Upper arms	71.7	6.6
	elbow 62.2	Lower arms	55.3	4.2
52.1	Hip	Hands	43.1	1.7
	wrist 46.2	Upper legs (thighs)	42.5	21.5
28.5	Knee			
		Lower legs	18.2	9.6
4.0	Ankle	Feet	1.8	3.4
			58.0	100.0



Example 7.12
Determine the position of the CM of a whole leg. Assume the person is 1.70 m tall.
a) when stretched out

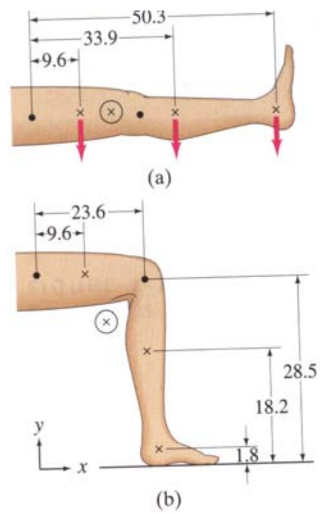


FIGURE 7–25 Example 7–12: finding the CM of a leg in two different positions (⊗ represents the calculated CM).

b) when bent at 90°

Newton's 2nd Law for a system of particles:

$$\mathbf{Ma_{CM} = F_1 + F_2 + F_3 = F_{net}}$$

Example 7.13

A rocket is shot into the air as shown. At the moment it reaches its highest point, a horizontal distance d from its starting point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair and falls vertically to Earth. Where does part II land? Assume g = constant.